Introduction

Fig. 8.1: The four stages of biosignal processing

Types of Signals

Fig. 8.2: Types of biological signals
Types of Signals

![Figure 8.3: Impulse series](image)

Analog-to Digital Conversion

**08.01 Sampling of Signals, How Often?**

Without exception, all biosignals are analog signals. Processing of biosignals by computers therefore requires discretization (i.e., sampling and quantification). This panel explains the sampling process without referring to formulas.

The **sampling theorem**, mathematically phrased by Shannon and Nyquist states that a signal must be sampled at a rate at least twice the rate of the highest-frequency component present in the signal. If we use a sampling rate that is too low, the signal is distorted. If we obey the sampling theorem, the complete *content* of the signal is retained. This is illustrated by the following example.

An EEG usually contains statistical, more or less sine-wave-shaped fluctuations that may occur at a rate of up to 30 times/second. This can also be expressed by saying that the EEG contains frequencies up to 30 Hz. Higher frequencies may also be present (e.g., from other signal sources) but these are generally not of semantic interest.

The sampling theorem then prescribes that we should sample the EEG at least at 2 x 30 = 60 Hz to keep all signal properties. Table 8.1 gives the frequency bands of interest and the most commonly used sampling rates for some biosignals. For instance, for ECGs (bandwidth, 0.15-150 Hz) a sampling rate above the Shannon frequency (300 Hz) is not what usual. If we obey the rule of the sampling theorem it is, in principle, possible to restore the original analog signal by digital-to-analog conversion.
Analog-to Digital Conversion

### 08.01 Bandwidths, Amplitude Ranges, and Quantization of Some Frequently Used Biosignals

<table>
<thead>
<tr>
<th>Signal</th>
<th>Bandwidth (Hz)</th>
<th>Amplitude range</th>
<th>Quantization (bits)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Electroencephalogram</td>
<td>0.2-50</td>
<td>600 µV</td>
<td>4-6</td>
</tr>
<tr>
<td>Electrocardiogram</td>
<td>0.2-15</td>
<td>10 mV</td>
<td>4-6</td>
</tr>
<tr>
<td>Electromyogram</td>
<td>0.15-150</td>
<td>10 mV</td>
<td>10-12</td>
</tr>
<tr>
<td>Electrooculogram</td>
<td>20-8000</td>
<td>10 mV</td>
<td>4-8</td>
</tr>
<tr>
<td>Blood pressure</td>
<td>0-60</td>
<td>400 mm Hg</td>
<td>8-10</td>
</tr>
<tr>
<td>Spirogram</td>
<td>0-40</td>
<td>10 L</td>
<td>8-10</td>
</tr>
<tr>
<td>Phonocardiogram</td>
<td>5-2000</td>
<td>80 dB</td>
<td>8-10</td>
</tr>
</tbody>
</table>

Table 8.1. Bandwidths, Amplitude Ranges, and Quantization of Some Frequently Used Biosignals.

### 08.02 Sampling of Signals: How Accurate?

When sampling a signal, we use an analog-to-digital converter (A-D converter or ADC). Samples are taken at a rate at least twice the rate of the highest frequency component contained in the signal (i.e., the mixture of signal plus noise, unless the noise has been filtered out beforehand), and the samples are quantitated and expressed as numbers. The latter is always done with a limited accuracy and may, in principle, add so-called quantization noise to the sampled signal. This quantization noise should generally not exceed the noise that is already present in the signal, or as expressed in more general terms, discretization by the ADC should not increase the information entropy (see Chapter 2); syntactic and semantic signal properties should be left intact.

The degree of quantization can be expressed as the number of quantization steps for the range of possible amplitude values. If the signal amplitude spans a range of $A$ volts (e.g., from $-A/2$ to $+A/2$) and the quantization step is $\Delta$, then the number of quantization steps is $m = A/\Delta$.

In practice, let $n$ be a power of 2: $n = 2^x$, so that the quantization of the ADC can be expressed in $n$ bits.

For instance, an ADC with an accuracy of 10 bits can discern $2^{10}$ different amplitude levels, resulting in a resolution of about 0.19%, expressed as a percentage of the signal range. An ADC that delivers samples with 8-bit accuracy ($2^8 = 256$ steps) is called an 8-bit ADC. A 4-bit ADC only determines the sign of the signal (or whether it is larger or lower than some threshold).

For most biosignals a 6- to 12-bit ADC is sufficient; a 12-bit ADC implies a resolution of 0.0059% (less than 0.02%), related to a signal-to-noise ratio which is far superior to that attainable with most signal transducers.
Analog-to Digital Conversion

![Figure 8.4: Effect of sampling frequencies](image)

**Figure 8.4:** Effect of sampling frequencies
How often?
E.g.: blood pressure in healthy, physical examination, cardiac surgery

Application Areas of Biosignal Analysis

![Figure 8.5: Four different situations in biosignal processing](image)

**Figure 8.5:** Four different situations in biosignal processing:
output signal, evoked signal, provocative test, process modelling.
Biosignal Processing Methods

Signal-Amplitude Properties

Figure 25.1: Amplitude distribution functions (density distribution function (ddf))
Signal-Amplitude Properties

Figure 25.2: Examples of 2D amplitude distributions (Lissajous Figures)

a) sine and cosine with same frequency and no phase difference
b) sine and cosine of twice the sine-wave frequency
c) sine and sine wave with amplitude taken half a period apart.

Signal-Amplitude Properties

Figure 25.3: Vectorcardiogram: the heart as an electric dipol.
Fig. 25.4: Two-dimensional amplitude ddfs for EEG amplitudes and RR intervals. Correlation functions.

Signal-Amplitude Properties

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**25.6 One-Dimensional Distributions, Means and Variances**

In many instances we want to follow how signal properties change as a function of time in order to detect trends in the underlying biological process. In that case we compute parameters for short observation periods, but in other instances we might be interested in changes over extended periods of time. The parameters that we compute depend on the signal to be examined, and denoted for each period $T$ parameter values that describe the signal in this period are derived parameter values for amplitude ddfs, which may not be a single number, which may be noisy or wide, and which may vary over time. If the signal can be described as:

$$x(t) = a(t) + n(t)$$

Then the sampled signal can be expressed as:

$$x_n = x(\frac{n}{f_s})$$

where $f_s$ is the sampling frequency. The amplitude ddf of $x_n$ can be expressed as:

$$E(x_n) = \sum_{i} f(x_i)$$

Another way of writing the mean value of the variable is:

$$E(x) = \frac{1}{N} \sum_{i} X_i$$

which is also called the first-order moments of the ddf $E(x)$. Another way of writing the mean value of the variable is:

$$E(x) = \frac{1}{N} \sum_{i} x_i$$

where $N$ is the observation period. $E(x)$ is equal to the mean of the signal. For ordinary signals, this equation holds for all observations. If the deterministic signals may vary, then a similar way it is possible to write for the variance of the signal (the second-order moments of the ddf):

$$E(x - E(x))^2 = \int_{-\infty}^{\infty} (x - E(x))^2 f(x) dx$$

For a signal with a mean value of zero ($E(x) = 0$), the variance becomes:

$$E(x^2) = \int_{-\infty}^{\infty} x^2 f(x) dx$$

The significance is the square root of the variance. The signal-to-noise ratio (SNR) of a signal that is the ratio of a signal plus noise, $x(t) = x(t) + n(t)$, measuring that $E(x)$ and $E(n)$ as equal to zero is defined as the ratio of the variances of signal and noise, $S$ and $N$, respectively:

$$SNR = S / N = \frac{E(x^2)}{E(n^2)}$$
**Frequency Spectra and Filtering**

Figure 25.5: Examples of three biological signals with their frequency spectra
Frequency Spectra and Filtering

Figure 25.6: Frequency components.

25.03 Wavelet Analysis

In Fourier analysis, a signal is thought to be composed of sines and cosines. By using the Fourier transform, a signal can be decomposed into these basic functions. The samples of the transformed signal (Fourier coefficients) represent the contribution of sine and cosine functions at different frequencies. A disadvantage of Fourier analysis is that it is difficult to compose a signal that is limited in time. By using functions that, by definition, stretch out into infinite time, it is therefore difficult for a Fourier function to approximate sharp changes in a signal. For example, a simple and time-limited signal, a spike, is decomposed by Fourier transformation into an infinite number of sines and cosines (see also Fig. 25.6a for the decomposition of a block signal).

A way to tackle this problem is through wavelet analysis. It uses the same principle as Fourier analysis, namely, that signals are composed of basic functions, called wavelets. The most important difference between these wavelets and the sines and cosines used in Fourier analysis is that wavelets are limited in time. The procedure for wavelet analysis is to choose a suitable wavelet prototype function (also called mother wavelet or analyzing wavelet) that meets certain constraints. All composing functions are derived by stretching or scaling the mother wavelet both in time and in amplitude. Using a wavelet transform, the signal is decomposed into these scaled versions of the mother wavelet. In fact, the composing cosines used in Fourier analysis can also be seen as stretched, scaled, and shifted versions of a mother-cosine. In Fourier analysis, the composing functions are infinite in the time domain because they represent exactly one frequency. In wavelet analysis, the composing wavelets have a limited extent both in the time domain and in the frequency domain, where contributions from frequencies outside a certain area are negligible.

The most important result of the wavelet transform is the location of the composing wavelets in time. Sharp, time-limited signal parts will be represented by wavelets that are scaled down in duration. As in Fourier analysis, the contribution of the composing wavelets to the signal provides information about the temporal properties of the signal on different time scales. Additionally, the locations of the composing wavelets provide information about the position of a specific signal property. Figure 25.2 shows two examples of a mother wavelet, both from the well-known Coiflet wavelet family.
Frequency Spectra and Filtering

Figure 25.7: Two examples of a mother wavelet.

Figure 25.8: Frequency spectra of an ECG.
Frequency Spectra and Filtering

**Figure 25.9:** Schematic representation of filters.

Frequency Spectra and Filtering

**Figure 25.10:** ECG and its band-pass filtered version.
Frequency Spectra and Filtering

15.01 Relationships between True Positive (TP), True Negative (TN), False Positive (FP), and False Negative (FN)

<table>
<thead>
<tr>
<th>Truth</th>
<th>TP</th>
<th>FN</th>
<th>100%</th>
</tr>
</thead>
<tbody>
<tr>
<td>+</td>
<td>a</td>
<td>b</td>
<td>a+b</td>
</tr>
<tr>
<td>-</td>
<td>c</td>
<td>d</td>
<td>c+d</td>
</tr>
</tbody>
</table>

Table 15.1. Relationships between True Positive (TP), True Negative (TN), False Positive (FP), and False Negative (FN).

15.02 Illustration of Sensitivity, Specificity, and Predictive Value

<table>
<thead>
<tr>
<th>Truth</th>
<th>+</th>
<th>-</th>
</tr>
</thead>
<tbody>
<tr>
<td>+</td>
<td>a</td>
<td>b</td>
</tr>
<tr>
<td>-</td>
<td>c</td>
<td>d</td>
</tr>
</tbody>
</table>

Table 15.2. Illustration of Sensitivity, Specificity, and Predictive Value (see text).

- Sensitivity = a/(a+b)
- Specificity = d/(c+d)
- PPV = a/(a+c) ... accuracy for positive prediction
- NPV = d/(b+d) ... accuracy for negative prediction

Frequency Spectra and Filtering

Figure 15.6: Distributions of systolic blood pressure of hypertensive and nonhypertensive people.
- a) Population survey
- b) Primary care
- c) Cardiac clinic
Frequency Spectra and Filtering

Figure 15.7: Distributions of the primary population of Fig. 15.6.

Figure 15.8: ROC curves of the population of Fig. 15.7.
Signal-to-Noise Ratio

Figure 25.11: Coherent averaging in an ECG recording.

In coherent averaging we compute the sum of, say, $K$ waveforms $s_0$, which are extracted after detection from a noisy signal $x(t) = s_0(t) + n_0(t)$. The original signal variance is $S_0 = \sigma_{s_0}^2$, and the noise variance is $N_0 = \sigma_{n_0}^2$, so that the SNR is:

$$\text{SNR}_0 = \frac{S_0}{N_0}.$$

The sum of the $K$ signal waveforms $s_0$ will result in a waveform $s_1$ which is $K$ times as large as the original waveform, that is, $s_1 = Ks_0$. The resulting signal dispersion is also $K$ times as large: $\sigma_{s_1} = K\sigma_{s_0}$. The variance of $s_1$ is then $K^2 \sigma_{s_0}^2 = K^2 S_0$.

We assume that the noise has a normal distribution. The $K$ noisy waveforms $n_0$ are also summed to a new noisy signal, $n_1$. It can be proven that the variance of $n_1$ is $K^2$ times as large as the variance of $n_0$, so that $\sigma_{n_1} = K\sigma_{n_0}$. The SNR after summation is then:

$$\text{SNR}_1 = \frac{S_1}{N_1} = \frac{K^2 S_0}{K^2 N_0} = \frac{K^2}{K^2} \frac{S_0}{N_0} = K \text{SNR}_0.$$

This implies that the SNR has improved linearly with the number of summed waveforms.
Signal Detection

### Table 25.1: Four Different Detection Situations for the Decision $D$ that an event $S$ is Present

<table>
<thead>
<tr>
<th>Situation</th>
<th>Description</th>
<th>$S$</th>
<th>$D$</th>
</tr>
</thead>
<tbody>
<tr>
<td>TP</td>
<td>The event is present AND is correctly detected</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>FP</td>
<td>The event is not present AND is incorrectly detected</td>
<td>0</td>
<td>1</td>
</tr>
<tr>
<td>TN</td>
<td>The event is not present AND is correctly not detected</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>FN</td>
<td>The event is present AND is incorrectly not detected</td>
<td>1</td>
<td>0</td>
</tr>
</tbody>
</table>

Table 25.1: Four Different Detection Situations for the Decision $D$ that an event $S$ is Present.

*TP, true positive; TN, true negative; FP, false positive; FN, false negative.

Figure 25.12: Detection and estimation may reinforce each other.
Signal Detection

Figure 25.12: An artificial signal of amplitude-modulated impulses.